BEST: International Journal of Humanities, Arts,
Medicine and Sciences (BEST: IJHAMS)
ISSN (P): 2348-0521, ISSN (E): 2454-4728
Vol. 3, Issue 10, Oct 2015, 169-176

A PHENOMENOLOGICAL METHOD FOR CALCULATING THE DOUBLE BACKBENDING FOR EVEN-EVEN RARE EARTH NUCLEI

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ABSTRACT

The Yrast band energies for some even rare earth nuclei (^{156}Dy, ^{158}Er, ^{160}Er, and ^{168}Hf) showing a double backbending. The theoretical calculations for these nuclei are investigated through band mixing in which the rotational ground state band is described by the variable moment of inertia model (VMI). The s-band was described through a linear equation of the angular momentum. We proposed a new interband coupling state equation in the form of a double Gaussian potential instead of single Gaussian potential between the last two bands. Our model using the interband interaction gives reasonable good well results for the available experimental data.

KEYWORDS: Earth Nuclei, VMI Model Formula, Double Backbending

I. INTRODUCTION

As has been known for more than three decades, deformed nuclei commonly show a rotational anomaly (known as backbending), where the yrast, collective band undergoes an abrupt increase in its moment of inertia (as a function of frequency, for example). In the rare earth region the first broken pair is from the neutron intruder orbital $1i_{13/2}$. The backbending is a relatively widespread phenomenon within the rare earth region, but only very few nuclei exhibit a second anomaly in the moment of inertia which is regarded as being caused by a successive breaking of a $1h_{11/2}$ proton pair. These second backbending was done by theoretical studies [1-3] based on the neighboring nuclei of the $N \approx 90$ isotopes. A semimicroscopic model inroduced to give a good discription for some nuclei having a second anomaly[4,5,6].Here In this paper we use a phenomenological method that was previously used by Bonatsos [7] in evaluating backbending and obtained accurate description of all the yrast, β and γ bands using the variable moment of inertia (VMI)[8] model. In addition, the superband state were described through a linear equation in terms of the angular momentum and the interaction strength between these two bands is taken as a single Gaussian form[9] or introducing a predicted shape in the form of a double Gaussian form. The paper is organized as follows: In Section 2, the method of calculations for the energy levels and backbending. The results obtained are presented in Section 3. Finally, Section 4 provides the findings and conclusions.

II. THEORETICAL METHOD OF CALCULATIONS

The energies of the yrast states or yrare states can be calculated as a result of crossing the ground state $E_g(I)$ band with the superband $E_s(I)$ through a certain interaction V(I), using the formalism [7]

$$W(I)=\frac{E_g(I)+E_s(I)}{2}\mp\left\{(\frac{E_g(I)-E_s(I)}{2})^2+V^2(I)\right\}^{1/2} \quad (2.1)$$

with the continuity condition

$$\frac{\partial W_{\pm}(I)}{\partial J(I)}=\frac{\partial E_g(I)}{\partial J(I)}=0 \quad (2.2)$$

The negative sign correspond to the yrast or β or γ bands and plus sign correspond to the next band (the yrare band if the lower band is the yrast band).

The well-known VMI model can describe the energy levels of the ground state rotational band for even-even nuclei as,

$$E_g(I)=\frac{I(I+1)}{2J(I)}+\frac{1}{2}C(J(I)-J_0)^2 \quad (2.3)$$

Which contains C and J_0 are the two parameters of the model. By applying (2.2) in. (2.3) we get the equilibrium value of the variable moment of inertia J (I) as:

$$J^3(I)-J_0J^2(I)-\frac{I(I+1)}{2C}=0 \quad (2.4)$$

This third order equation may be solved algebraically for real values of J(I). The superband is described by a linear equation is a function of angular momentum in the form

$$E_s(I)=E_0+A[I(I+1)-K^2] \quad (2.5)$$

With K=0 or 1. In these equations (2.4 and 2.5) E_0 and A are two parameters obtained from the fitting process.

Since the exact form of the interband interaction is not precisely known, so a single and a double Gaussian shape of the interaction band can be used as a trial for calculations as a function of the angular momentum t in the follwing forms

i- For a single Gaussian we used,

$$V(I)=V_0\,e^{-x(I-I_0)^2} \quad (2.6)$$

With V_0 and x are two parameters obtained from optimization to the experimental data:

ii- for the double Gaussian we used,

$$V(I)=V_{01}\,e^{-x1(I-I_{01})^2}+V_{02}e^{-x2(I-I_{02})^2} \quad (2.7)$$

With $V_{01}, V_{02}, x_1, and\ x_2$ are four parameters obtaind from fitting process. $I_{01}\ and\ I_{02}$ are two chosen values of the angular momentum at the first and second crossing region.

When the yrast levels are fitted, the energy equation (2.1) have six/eight independent parameters (two for the ground state band, two for the superband and two/four for the interband interactions. These parameters are obtained by

fitting six/eight experimental energy levels of the yrast band. Usually the first two energy levels for the ground state band, two energy levels after band crossing and two near the first band crossing point to use (2.6) and two more near the second band crossing point to use (2.7) are chosen. The first two levels of the yrast band fix the value of the VMI parameters (J_0 and C). The two levels after the band crossing determine the superband parameters (E_0 and A). Two/four levels near the band crossing point fixes the interband interaction parameter $V_{01},\ x_1, V_{02}\ , and\ x_2$.

For the backbending calculation we have used the moment of inertia $J(I)$ according to the following expression:

$$\frac{2J(I)}{\hbar^2} = \frac{4I-2}{E(1+2)-E(I)} \tag{2.8}$$

and the angular frequency is defind as

$$\hbar\omega = \frac{dE(I)}{dI} \approx \frac{1}{2}[E(I+2) - E(I)] \tag{2.9}$$

III. RESULTS AND DISCUSSIONS

The rotational motion of a nucleus (which is considered as a rigid rotor) the moment of inertia of the different energy states should be constant as we go to higher excitation enregy[10]. But really if the moment of inertia is not constant but increases gradually as we go to more rapidly rotating states. This effect known classically as "centrifugal stretching" would not occur for a rigid rotor but would occur for a fluid. Because the rotating nuclei have moments of inertia somewhere between that of a rigid rotor and of a fluid, it is not surprising that centrifugal stretching occurs. So there is a more instructive way to plot the data on the rotational structure. So, by plotting the relation between $(\hbar\omega)^2$ vs $(\frac{2J(I)}{\hbar^2})$, there appears a gradual increase in the moment of inertia in the lower energy states, then a radical change in behavior and then return again to the gradual increase once more. This effect is known as a first backbending which is ocucure in some heavy nuclei due to the breaking of couple pair of nucleon. When this effect occurs, the unpaired nucleons go into different orbits and change the moment of inertia. Sometimes this happeins again at heigher energy states to give the second backbending as we can see in our calculations.

There are few nuclei in the rare earth region which exhibit a second anomaly in the moment of inertia in the yrast band. Using the equations presented in the previous section we have fitted experimental data for even-even (^{156}Dy, ^{158}Er, ^{160}Er, and ^{168}Hf) nuclei. These nuclei presents a second anomaly, given the fact that we deal with neutrons from the $\nu i_{13/2}$ intruder orbital and protons from $\pi h_{11/2}$ intruder orbital, which are responsible for the first and the second band crossing, respectively. These nuclei are classified according to two main collective modes, namely, transitional and rotational which is considered as a strongly deformed nuclei according to there R- ratio R= $\frac{E(4)}{E(2)}$ which is found in the interval of 2.9~3.2.

The experimental data for the yrast energy states of these nuclei with positive parity and spin up to I^{π} = 42^+, 36^+, 54^+,and 48^+ for (^{178}Dy, ^{158}Er, ^{160}Er, and ^{168}Hf) [11-14] respectively. Admittedly, introducing (2.7) in calculations for the yrast band energy states gives a satisfactory results than that calculated by introducing (2.6). This is shown in figure 1 (a) which presents the comparison between the theoretical results of the energy states (closed filled circles) using (2.7) compared to the experimental data (filled rectangles) for ^{156}Dy nucleus.

In figure 1 (b) the theoritical calculation using (2.7) for the yrast band energy states of ^{158}Er gives a quit well fit

with the experimental data at low and high energy states, while figure 1 (c) give the results for ^{160}Er energy states in which at low energy states gives a well reproduced results but do not give a very well results at high energies. As can be seen in figure 1 (d) ^{168}Hf the calculations for the yrast band energy states gives a good results for the calculated energies using (2.7) compared to the experimental data. The parameters used in fitting procedure tabulated in table 1 and the calculated energy values compared to the experimental yrast energy states are presented in table 2.

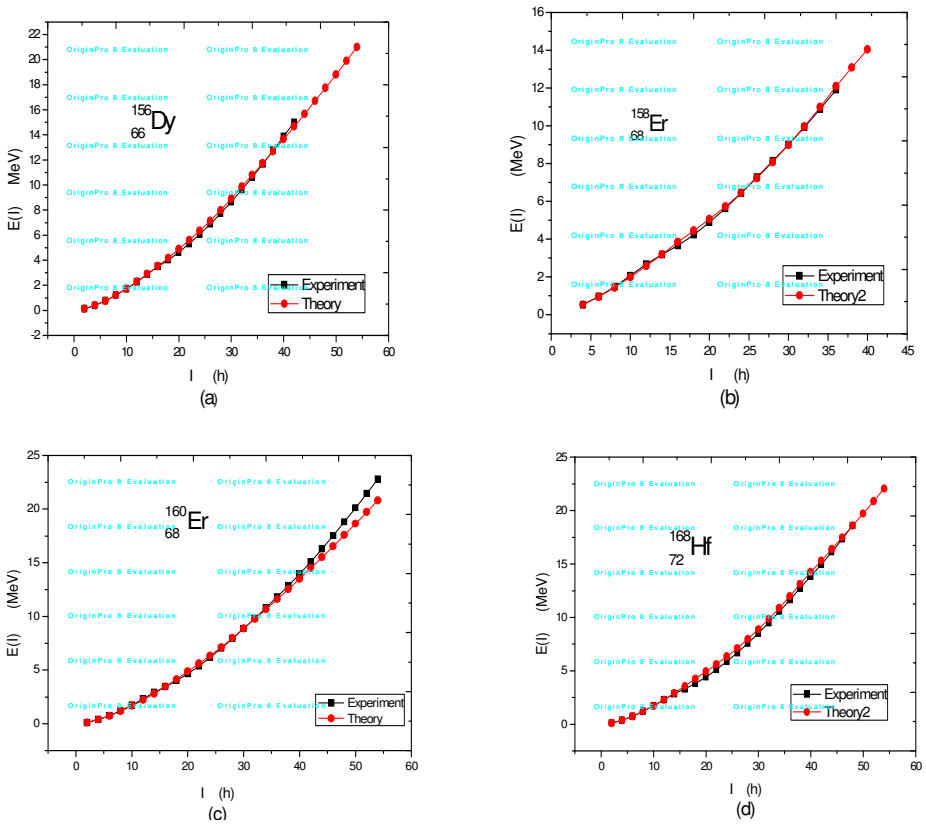
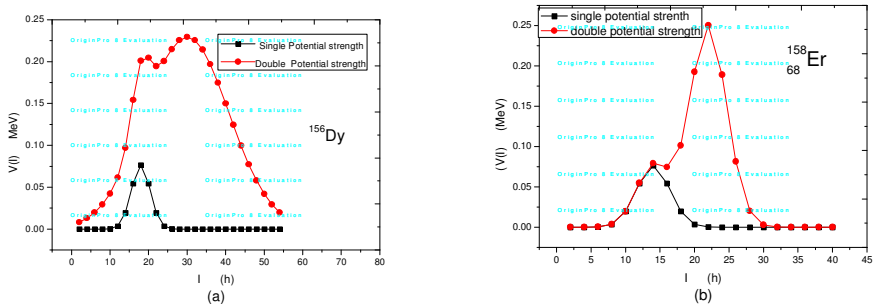


Figure 1: (A,B,C,D): Presents the Comparison between the Theoretical Results (Closed Filled Circles) Using Eq.(2.7) Compared to the Experimental Data (Filled Rectangles) for Inset Nuclei. the Yrast Energy States for These Nuclei are Available in Ref. [11-14]

The interaction strength of (2.6) and (2.7) is presented in figure 2 (a,b,c,d,e) for the single gaussian potential strength (filled rectangles) compared to that for the double gaussian potential strength (filled circles). It is clear from the figure's that the height strength of the second peak of the double interaction potential strength is much heigher than that of the single potential strength in the order of about four times. These



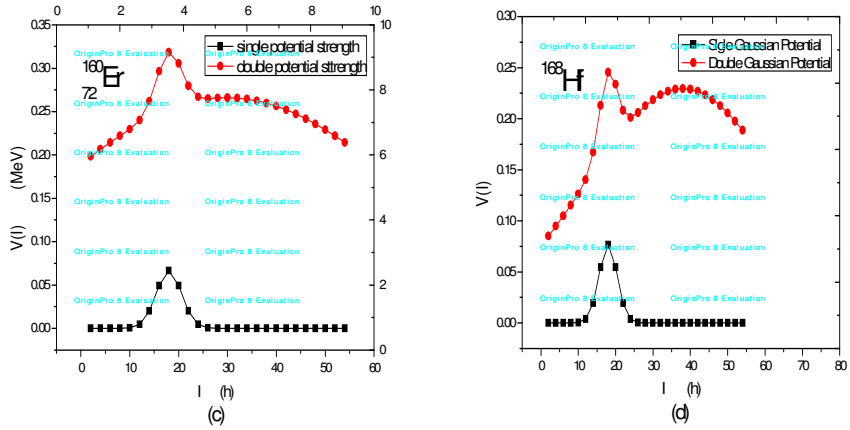


Figure 2(A,B,C,D,): Represents the Interaction Strength for Single Gaussian Potential Strength Eq. (2.6) (Filled Rectangles) and for the Double Gaussian Potential Strength Eq.(2.7) (Filled Circles)

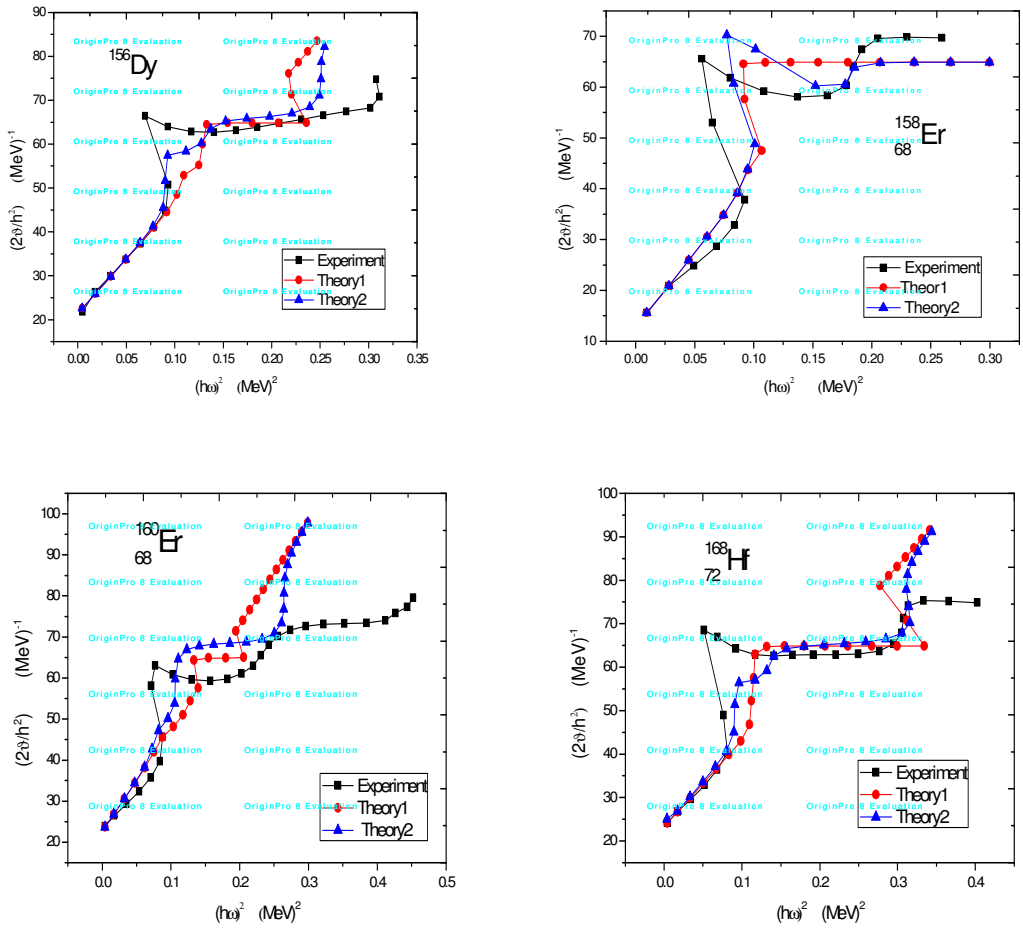


Figure 3 (A,B,C,D): The Backbending Plot of this Nucleus ^{156}Dy, which gives Satsfactory Fit with the Use of (2.7) (Theory2)with Filled Triangles for the First Backbending and Second Upbending than that Obtained by Using of Equ.(6) (Theory1) with Filled Circles Compared to the Experimental Data(Experiment) with Filled Triangles

For the better understanding of the mutiple backbending phenomena, the theoretical results and experimental data are calculated using Eq.(2.8) and Eq.(2.9) by means of backbending plots i.e the ($\frac{2J(I)}{\hbar^2}$) vs $(\hbar\omega)^2$. In the backbending plots of the four chosen nuclei, it is obvious that the zigzag shape is approximatly well reproduced with the use of (2.7) than

(2.6). It is well known that the backbending plots for the first and the second backbending which for some extent well reproduced in calculations but not the same like the energy levels. This happens because as we know that the backbending curves depends on the energy difference between consecutive states and moreover through a quadratic angular frequency $(\hbar\omega)^2$ which is sensitive to small deviations. So, It is obvious that the backbending is well reproduced quit well for the ^{156}Dy, ^{158}Er and ^{168}Hf but it is acceptetable in case of ^{160}Er especially for the second anomaly. An especially good agreement is found for moderate spin state at the first backbending. The second observed moment of inertia anomaly is really not considered as a second backbending but a relatively weak upbending and the calculations supposed to be less pronounced compared to the experimental data than the first one.

Finally, the theoretical study with the standard VMI model for the yrast band using Bonatsos method which assumes that there are two band crossing with interband interaction potential in the form of single gaussian potential shape gives a satisfactory results for the nuclei with one band crossing only (i.e first backbending) [9]. But in case of two band crossing (i.e double backbending) it not give very well acceptable to use a double gaussian potential in the theoretical calculations to fit the double zigzag in the moment of ineria of some even- even rare earth nuclei.

Table 1: Comparison between Our Results (Etheo) and Different Experimental (Eexp) [11-14] for the Yrast Energies For ^{156}dy, ^{158}er, ^{160}er, and ^{168}hf Isotopes). All the Energy Values are in Mev

Nucleus	^{156}Dy		^{158}Er		^{160}Er		^{168}Hf	
	E_{exp}	E_{theo}	E_{exp}	E_{theo}	E_{exp}	E_{theo}	E_{exp}	E_{theo}
2^+					0.126	0.127		
4^+					0.390	0.389	0.124	0.124
6^+	0.138	0.133			0.766	0.748	0.386	0.386
8^+	0.404	0.403			1.229	1.184	0.757	0.753
10^+	0.771	0.772	0.192	0.192	1.761	1.682	1.214	1.203
12^+	1.216	1.217	0.527	0.527	2.340	2.231	1.736	1.721
14^+	1.725	1.725	0.970	0.952	2.933	2.823	2.306	2.298
16^+	2.286	2.285	1.494	1.443	3.467	3.467	2.858	2.925
18^+	2.888	2.892	2.073	1.989	4.022	4.152	3.310	3.588
20^+	3.499	3.531	2.681	2.576	4.663	4.868	3.833	4.258
22^+	4.026	4.193	3.191	3.193	5.385	5.615	4.440	4.936
24^+	4.636	4.899	3.663	3.846	6.178	6.346	5.124	5.619
26^+	5.320	5.617	4.230	4.454	7.032	7.132	5.875	6.345
28^+	6.070	6.345	4.888	5.058	7.933	7.981	6.687	7.132
30^+	6.878	7.132	5.629	5.721	8.870	8.887	7.562	7.980
32^+	7.739	7.980	6.435	6.446	9.831	9.770	8.501	8.890
34^+	8.651	8.890	7.280	7.232	10.82	10.674	9.500	9.861
36^+	9.611	9.861	8.139	8.081	11.83	11.601	10.55	10.894
38^+	10.62	10.800	9.014	8.990	12.87	12.549	11.64	11.989
40^+	11.67	11.733	9.920	9.962	13.96	13.518	12.74	13.146
42^+	12.77	12.688	10.88	10.995	15.09	14.506	13.85	14.263
44^+	13.89	13.662	11.90	12.089	16.28	15.513	14.97	15.316
46^+	14.99	14.656			17.52	16.538	16.13	16.391
48^+					18.81	17.581	17.34	17.485
50^+					20.11	18.641	18.61	18.599
52^+					21.44	19.718		
54^+					22.79	20.812		

Table 2: the Fitted Parameters for the Four Nuclei are Listed

Nucleus	$J0_1$ (Mev)$^{-1}$	C (Mev)^3	E0 (Mev)	A (Mev)	V01 (Mev)	X1	V02 (Mev)	X2
^{156}Dy	21.18	0.00198	1.72	0.00771	0.0765	0.085	0.2295	0.050
^{158}Er	12.48	0.00167	1.82	0.00771	0.0765	0.085	0.2500	0.070
^{160}Er	22.38	0.00198	1.72	0.00771	0.0765	0.085	0.4000	0.005
^{168}Hf	23.18	0.00249	1.72	0.00771	0.0765	0.085	0.2295	0.009

IV. CONCLUSIONS

The R-ratio analysis indicate that the even-even nuclei ^{156}Dy, ^{158}Er, ^{160}Er, and ^{168}Hf lies in the transitional and rotational regions. These nuclei characterized by a strong deformation. It have a second anomaly (i.e second upbending) in the experimental data. The energy levels and backbending calculations are carried out using the VMI model formula in the ground state band, and a simple rotational formula depends on the angular momentum and characterized by a constant moment of inertia for the s-band. The interaction potential between these two bands is not well known till now so we introduce a new formula for the interband interaction in the form of a double gaussian potential instead of a single gaussian one. The proposed formula gives an accurate results for the energy levels of the chosen nuclei but it gives a reasonable results for first backbending and a second upbending calculations. This emphasis that the interband interaction potential do not have a fixed shape in the backbending calculations and can be predicted for the double backbending for nuclei in the rare earth region.

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